Cities under Stress[*]

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Abstract

This paper studies the effect of terrorist attacks on the internal structure of cities. We develop an urban framework with capital structures suitable for the study of this question and analyze the long and short term implications of this type of events. In the long run, the analysis shows that a terrorist attack will affect urban structure only modestly, relative to the potentially large decrease in the level of economic activity in the city. Land rents will not decline at all locations. In the short run, agglomeration forces will amplify the effect of the original destruction and will reduce urban economic activity temporarily.

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1. INTRODUCTION

This paper analyzes the effect of terrorism on the internal structure of cities. Terrorist attacks are urban shocks that affect the structure of a city both temporarily and permanently. They result in changes in urban employment and residential densities,

buildings and structures, and the location of business and residential areas, as well as in changes in wages and land rents, that we seek to study here.

There are two main dimensions in which terrorist attacks affect urban structure. The first one is the actual destruction of physical capital in certain parts of the city caused by a realized shock. The second, with potentially larger effects, is the change in the expectation of residents and firms of the likelihood of future attacks after a terrorist act is realized. The change in expectations distorts investment decisions permanently. The threat of future attacks may also alter the costs of commuting, transporting goods, and congestion, as well as the benefits of agglomeration. Modern terrorism is characterized by the potential for attacks to create devastation on an unprecedented scale and by the fact that this extreme threat is likely to persist indefinitely. The objective of this study is to investigate how the structure of cities might respond to such extreme actual and threatened shocks.

Our starting point for analyzing the consequences of terrorist attacks is the equilibrium model in Lucas and Rossi-Hansberg (2002), where the distribution of business and residential land, wages, and land rents, are the result of the trade-off between spatial production externalities and commuting costs. This theory allows us to study how different urban structures, like London (with one Central Business District), New York (with two CBD's), or Los Angeles (with multiple CBD's), will be affected differently by terrorism.

A crucial component of terrorist attacks is the implied destruction of buildings and infrastructure. To address the destruction of physical capital we add structures to the model in Lucas and Rossi-Hansberg (2002) and embed it in a dynamic framework[1]. We then use the model to study the steady state implications of changes in the expectation of future attacks. Transitional effects may also be substantial, hence, we

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[1]A starting point is also the model in Berliant, Peng and Wang (2002) that introduces capital to the constant density framework in Fujita and Ogawa (1982).

introduce a formulation of the problem that includes costs of adjustment in capital investments. These adjustment costs imply that, in our theory, cities will recover slowly from realized attacks. The analysis considers the nature of such transitions.

The threat of future attacks leads to permanent changes in city structure since the potential of future physical capital destruction reduces the returns to capital investments in many locations within the city. Firms and residents will be willing to invest less in structures at those locations, which in turn changes their bid rents, and therefore the structure of the city. This new structure may benefit some areas in favor of others thereby increasing the returns to capital and land rents in some of them. That is, the effect of a terrorist threat on a city does not necessarily imply uniformly lower land prices. In the classic monocentric model, where the structure of the city is given by assumption, terrorist threats will lower land rents uniformly. In our framework we can also show that a terrorist threat will decrease the range of installed capital levels across locations within the city, and will reduce residential capital investments. Production, wages and population in the city will decrease. However, in all the numerical exercises presented, consumption and residential land rents in the city increase Agents compensate with consumption and space their lower investment in residential structures, given the reservation utility they require to stay in the city.

A realized terrorist attack will destroy capital structures that will be rebuilt in the future. The cost of rebuilding these structures may be considerable, it takes time, and distorts agglomeration forces and the structure of a city temporarily. In the absence of adjustment costs, the city will immediately return to the steady state capital stock in all areas of the city. The cost of the attack will only be the new investment necessary to rebuild the damaged structures. With adjustment costs, transition costs will be larger since the lack of capital will imply a reduction in agglomeration forces, and population in the city, that will result in a lower return to capital than in steady state.

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This implies that the transition will be more costly than the capital that was destroyed in the attack. We present a numerical example to illustrate the characteristics of this transition.

The threat of terrorist attacks increases transport and commuting costs as agents experience increased security checks and feel exposed to greater personal risk when in transit. Urban models are very sensitive to changes in commuting costs; small increases in these costs may lead to significant decreases in employment densities and therefore in TFP. Note that the indefinite nature of the extreme terrorist threat has the potential to cause significant adjustments in city structure, even without an actual attack, but especially if a major attack is realized somewhere in the world. We show that higher commuting costs result in lower output, smaller cities, less concentrated business areas, and a lower average capital level. In the numerical exercises presented, the proliferation of small business centers is one of the outcomes of higher commuting costs.

The threat of a terrorist attack also affects the net benefits of agglomeration along several dimensions. First, the net benefits to firms and agents of locating in a city decrease because of the threat to life and property. Second, since high-density areas are the most attractive target, the benefits to firms of locating near other firms decrease. Third, informal interactions among agents may be reduced too, due to the increased commuting costs. In short, the threat of terrorist attacks decreases the net benefits of interacting with other people, and therefore knowledge spillovers in a city. The result is a city with less concentrated employment and potentially more business centers. We will argue, however, that changes in the spatial scope of knowledge spillovers have only modest effects on urban structure.

The analysis will shed light on government policies that may reduce the transition costs of an actual attack, or avoid the long term effects of a terrorist threat. Given that an actual attack can change the expectation of the likelihood of future attacks

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and thereby move the city permanently to a worse equilibrium, is there a case for a temporary infusion of external resources to prevent such an outcome? The reaction of agents to the positive probability of future attacks is optimal. The danger that capital structures may be destroyed in the future implies that agents *should* invest less in those structures. If the fundamental parameters of our theory are not altered by the terrorist threat, we can show that the government can prevent the long term effect of a terrorist threat using a location specific investment subsidy. This is natural since the terrorist threat affects the city *only* by reducing incentives to invest. The production externality is on labor –and not on capital– and so the private investment decision is optimal given local productivity levels. Since the investment decision of private agents is not distorted, the analyzed subsidy is optimal only if the government has better information than the private sector about the likelihood of a future shock and can not credibly share it with private agents. We show that the gain in total urban production of implementing this policy is larger than the fiscal cost, if the government is right about the true probability of actual attacks.

Shortly after September 11, several papers analyzed the potential effect that such a horrible event would have on New York City and other cities in the US and the world. Glaeser and Shapiro (2002) conclude that the effect of September 11 on urban structure would be small, since, in the past, terrorism and wars have had a small effect on urban form. Their interesting paper enumerates and analyzes different potential effects of these attacks. Nevertheless, the paper leaves aside an effect that will play a central role in our study, namely, the effect of a terrorist threat on the returns to investments in structures. Mills (2002) does consider this effect and describes intuitively how the attack may lead to lower CBD land values and lower office building heights. The main forces he describes are present in our analysis and this conclusion is in line with our results. Wildasin (2002) describes the effect on local public finances of this type of shocks. All of these papers analyze different aspects of terrorist attacks

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on urban characteristics by enumerating and evaluating different forces in play, but lack an analytical framework to study them in detail. Harrigan and Martin (2002) study the resilience of cities to terrorist shocks with a simple analytical framework. Their analysis contributes to our understanding of the average effect of terrorism on urban form but is not designed to study allocations and prices within cities. The framework proposed in this paper is designed to analyze the spatial nature of terrorist attacks in a dynamic urban model. These shocks affect directly only particular areas in a city, but have important short and long term implications for the city as a whole.

The rest of the paper is structured as follows. Section 2 presents the model and several analytical results. Section 3 designs the capital subsidy that eliminates the long term effect of a terrorist threat and discusses other policies. Section 4 illustrates, with numerical exercises, the effect of terrorist shocks on steady state allocations in the case without adjustment costs, and the transition of a city that received a shock in the case of adjustment costs. Section 5 concludes.

2. THE MODEL

A model suitable for analyzing the effect of terrorist attacks on urban structure should include capital structures, and the dynamic accumulation decision to invest in them. The model should also allow for the analysis of localized attacks on particular areas within cities and their effect on land rents in these and other urban locations. Finally, we believe that such a framework should have endogenous densities so as to analyze the potential decreases in employment concentration and the implied effects on productivity and production levels. In this section we present a model with these characteristics.

We model a linear city with size $2S$. The boundaries of the city are located at S and $-S$ and its center at 0. A location $s \in [-S, S]$ is assumed to have a unit density of available land. Land is initially owned by absentee landlords that live and work

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outside the city.

2.1 Firm problem

Firms produce a tradeable good with unit price. Production per unit of land at location r at time t is given by

$$x_t(r) = g\left(z_t\left(r\right)\right) f\left(n_t\left(r\right), k_t\left(r\right)\right), \tag{1}$$

where $z_t\left(r\right)$ represents productivity, $n_t\left(r\right)$ employment per unit of business land, and $k_t\left(r\right)$ capital per unit of business land.

A firm at time t in location r, with a stock of capital given by $k_t\left(r\right)$, maximizes it's output minus labor and investment costs,

$$R\left(k_t\left(r\right), z^t\left(r\right), w^t\left(r\right)\right) \tag{2}$$
$$= \max_{\{n_j, k_{j+1}\}_{j=t}^{\infty}} \sum_{j=t}^{\infty} \beta^{j-t} \left[g\left(z_j\left(r\right)\right) f\left(n_j\left(r\right), k_j\left(r\right)\right) - w_j\left(r\right) n_j\left(r\right) - I\left(i_j^B\left(r\right)\right)\right]$$

$$\text{subject to } k_{j+1}\left(r\right) = \left(1 - \delta\right) k_j\left(r\right) + i_j^B(r) \text{ all } j, \tag{3}$$
$$\text{given } k_t\left(r\right)$$

where $i_t^B(r)$ denotes investments by firms, $z^t\left(r\right)$ and $w^t\left(r\right)$ the paths of productivity and wages at location r from t to infinity, and $I\left(\cdot\right)$ is an increasing and convex function for $i > 0$. One example is $I\left(i\right) = i + i^2$. The first part of the function represents the cost of capital, which is expressed in terms of the numerare good. The second part represents the adjustment costs. This implies that firms adjust their capital stocks slowly and results in some transitional dynamics. We assume that firm can borrow and lend at an interest rate $b = \left(1 - \beta\right)/\beta$. $R\left(k_t\left(r\right), z^t\left(r\right), w^t\left(r\right)\right)$ is the maximum amount a firm is willing to pay for land at location r. If a firm moves away from a particular location, it can always sell it's land together with the installed capital to

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a new firm or resident that wants to locate there.[2]

The first order conditions of this problem are (dropping in the notation the dependence on r)

$$g(z_j) f_n(n_j, k_j) = w_j, \tag{4}$$

$$\beta [g(z_{j+1}) f_k(n_{j+1}, k_{j+1}) + (1-\delta) I'(k_{j+2} - (1-\delta) k_{j+1})] = I'(k_{j+1} - (1-\delta) k_j)$$

$$\text{for all } j = t, t+1, \ldots \tag{5}$$

The last condition can be rewritten as

$$\sum_{j=s}^{\infty} \frac{((1-\delta)\beta)^{j+1-s}}{(1-\delta)} g(z_{j+1}) f_k(n_{j+1}, k_{j+1}) = I'(k_{j+1} - (1-\delta) k_j) \tag{6}$$

$$\text{for all } s \geq t.$$

Denote by $\hat{n}_j(k_t, z^t, w^t)$ and $\hat{\imath}_j^B(k_t, z^t, w^t)$, the employment density and investment sequences at location r that satisfy the first order conditions above, plus the necessary transversality condition

$$\lim_{j \to \infty} \beta^j [g(z_{j+1}) f_k(n_{j+1}, k_{j+1}) + (1-\delta) I'(k_{j+2} - (1-\delta) k_{j+1})] k_{j+1} = 0 \tag{7}$$

for all z^t bounded. Then, under suitable assumptions we present below, there exists a steady state capital level $\bar{k}(\bar{z}(r), \bar{w}(r))$ such that

$$\frac{\beta g(\bar{z}) f_k(\hat{n}(\bar{k}, \bar{z}, \bar{w}), \bar{k})}{1 - \beta + \delta\beta} = I'(\delta\bar{k}). \tag{8}$$

If for every set of values $(\bar{z}, \bar{w})$, for every r, there is a $\bar{k}(\bar{z}(r), \bar{w}(r))$ that satisfies the above expression (and a corresponding steady state expression for residential capital

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[2] All firms are identical ex-ante. Hence, if a firm wants to move to a different location at some future date it will sell its installed capital structures to an identical firm that will make identical decisions on investments. Therefore, the problem in (2) is, in equilibrium, equivalent to a problem in which we consider future reallocation explicitly.

that we will study below), then there is a steady state of this economy following the proofs in Lucas and Rossi-Hansberg (2002). In steady state, for all r,

$$R\left(\bar{k}, \bar{z}, \bar{w}\right) = \frac{1}{1-\beta}\left[g\left(\bar{z}\right)f\left(\hat{n}\left(\bar{k}, \bar{z}, \bar{w}\right), \bar{k}\right) - \bar{w}\hat{n}\left(\bar{z}, \bar{w}\right) - I\left(\delta\bar{k}\right)\right]. \qquad (9)$$

Given the nature of this problem, standard results imply that we can express it in recursive form, namely,

$$R\left(k_t, z_t, w_t\right) = \max_{n_t, k_{t+1}}\left[g\left(z_t\right)f\left(n_t, k_t\right) - w_t n_t - I\left(i_t^B\right) + \beta R\left(k_{t+1}, z_{t+1}, w_{t+1}\right)\right] \qquad (10)$$

subject to (3)

and all the standard results in Stokey, Lucas and Prescott (1989) Chapter 4 apply.

2.2 Consumer problem

Agents get utility out of consuming goods (c_t) and residential services (that consist of land (ℓ_t) and capital invested in that land (k_t)) every period according to a utility function $U\left(c_t, \ell_t, k_t\right)$. Agents live and work in the city if they get a lifetime future utility of at least $\bar{u}$. The problem of a consumer at time t, that lives at location r with capital $k_t\left(r\right)$, is given by

$$\min_{\{c_j, \ell_j, k_{j+1}\}_{j=t}^{\infty}}\sum_{j=t}^{\infty}\beta^{j-t}\left[c_j\left(r\right) + Q_j(r)\ell_j\left(r\right) + I\left(i_j^R\left(r\right)\right)\right] \qquad (11)$$

subject to

$$\bar{u} \leq \sum_{j=t}^{\infty}\beta^{j-t}\left[U\left(c_j\left(r\right), \ell_j\left(r\right), k_j\left(r\right)\right)\right], \qquad (12)$$

$$k_{j+1}\left(r\right) = k_j\left(r\right)\left(1-\delta\right) + i_j^R(r) \text{ all } j, \qquad (13)$$

given $k_t\left(r\right)$

where $Q_t(r)$ is the residential land rent, and $i_j^R(r)$ denotes residential investments.[3] Notice that the above problem assumes that agents can borrow and lend at an interest

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[3] As in the firm's problem, we do not need to consider future reallocations explicitly since all agents are identical.

rate b such that $\beta\,(1+b)=1$. The density of workers at location r and time t is given by

$$N_t\,(r)=\frac{1}{\ell_t\,(r)}. \tag{14}$$

The first order conditions for this problem are given by (dropping in the notation the dependence on r)

$$\zeta U_c\,(c_j,\ell_j,k_j) \;=\; 1, \tag{15}$$

$$\zeta U_\ell\,(c_j,\ell_j,k_j) \;=\; Q_j, \tag{16}$$

$$\beta\left[\zeta U_k\,(c_{j+1},\ell_{j+1},k_{j+1})+(1-\delta)\,I'\,(k_{j+2}-(1-\delta)\,k_{j+1})\right] \;=\; I'\,(k_{j+1}-(1-\delta)\,k_j)\,,$$

$$\text{for all } j \;=\; t,t+1,... \tag{17}$$

Again we can rewrite the last condition as

$$\sum_{j=s}^{\infty}\frac{((1-\delta)\,\beta)^{j+1-s}}{(1-\delta)}\zeta U_k\,(c_{j+1},\ell_{j+1},k_{j+1}) \;=\; I'\,(k_{j+1}-(1-\delta)\,k_j) \tag{18}$$

$$\text{for all } s \;\geq\; t.$$

Denote by $\hat{c}_j\,(k_t,Q^t)\,,\hat{\ell}_j\,(k_t,Q^t)\,$, and $\hat{\imath}_j^R\,(k_t,Q^t)$, for all t, the sequences of consumption, land, and investment that solve the above conditions given the initial capital at location r and the *sequence* of land rents, $Q^t(r)=\{Q_t(r),Q_{t+1}(r),...\}$, as well as the necessary transversality condition

$$\lim_{j\to\infty}\beta^j\left[\zeta U_k\,(c_{j+1},\ell_{j+1},k_{j+1})+(1-\delta)\,I'\,(k_{j+2}-(1-\delta)\,k_{j+1})\right]k_{j+1}=0. \tag{19}$$

Agents solve the above problem given a sequence of land rents $Q^t(r)$. How much would an agent be willing to pay for land at location r given that he requires utility $\bar{u}$ to stay in the city and can spend $w_t\,(r)$ at time t if he lives at location r? The sequence of bid land rents, $\hat{Q}^t(r)$, of an agent at location r solves the problem

$$\sum_{j=t}^{\infty}\beta^{j-t}w_j(r)=\sum_{j=t}^{\infty}\beta^{j-t}\left[\hat{c}_j\left(k_t,\hat{Q}^t\right)+Q_t\hat{\ell}_j\left(k_t,\hat{Q}^t\right)+I\left(\hat{\imath}_j^R\left(k_t,\hat{Q}^t\right)\right)\right]. \tag{20}$$

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Denote the solution to the problem above (if there are many choose the one with the highest present value), the sequence of bid land rents, by $\hat{Q}^t\left(k_t\left(r\right), w^t\left(r\right)\right) = \left\{\hat{Q}_t(k_t\left(r\right), w^t\left(r\right)), \hat{Q}_{t+1}(k_t\left(r\right), w^t\left(r\right)), ...\right\}$. Since agents can borrow and lend at an interest rate b that corresponds to their intertemporal discounting, we know that agents are indifferent between paying the present value of rents today or paying rent every period. The value of land at location r, the amount an agent would be willing to pay to *buy* that piece of land, is given by

$$q\left(k_t\left(r\right), w^t\left(r\right)\right) = \sum_{j=t}^{\infty} \beta_j^{j-t} \hat{Q}\left(k_t\left(r\right), w^t\left(r\right)\right). \tag{21}$$

Under suitable assumptions there exist, for all r, a steady state capital level, $\bar{k}\left(\bar{w}(r)\right)$, such that

$$\frac{\beta}{1-\beta+\delta\beta}\left[\frac{U_k\left(\hat{c}\left(\bar{k}, \hat{Q}(\bar{k}, \bar{w})\right), \hat{\ell}\left(\bar{k}, \hat{Q}(\bar{k}, \bar{w})\right), \bar{k}\right)}{U_c\left(\hat{c}\left(\bar{k}, \hat{Q}(\bar{k}, \bar{w})\right), \hat{\ell}\left(\bar{k}, \hat{Q}(\bar{k}, \bar{w})\right), \bar{k}\right)}\right] = I'\left(\delta\bar{k}\right). \tag{22}$$

In steady state, bid land rents solve,

$$\frac{\bar{w}}{1-\beta} = \frac{\hat{c}\left(\bar{k}, \hat{Q}\right) + Q\hat{\ell}\left(\bar{k}, \hat{Q}\right) + I\left(\delta\bar{k}\right)}{1-\beta}, \tag{23}$$

and the willingness to pay to buy land is given by

$$q\left(\bar{k}, \bar{w}\right) = \frac{\hat{Q}\left(\bar{k}, \bar{w}\right)}{1-\beta}. \tag{24}$$

Again there is a standard recursive representation of this problem of the form,

$$W(k_t, \hat{Q}_t) = \min_{c_t, \ell_t, k_{t+1}} \left[c_t + \hat{Q}_t\ell_t + I\left(i_t^R\right) + \beta W(k_{t+1}, \hat{Q}_{t+1})\right] \tag{25}$$

subject to (12) and (13).

2.3 Land Use

The total amount that a firm at r at time t would be willing to pay for one unit of land with $k_t\left(r\right)$ units of capital is given by $R\left(k_t\left(r\right), z^t\left(r\right), w^t\left(r\right)\right)$. Similarly, the

total amount that agents would be willing to pay for one unit of land at time t and location r if there are $k_t(r)$ units of capital installed is given by $q(k_t(r), w^t(r))$. The price of land includes the price of the installed capital. If a firm or a resident decides to move out of a particular location, they will sell the capital to a new firm or resident respectively. We assume that capital is not specific to the business or residential sector, so installed capital can be sold with land to agents in the other sector. Business and residential land prices depend on the past capital installed because it is costly to install capital and land prices include the stock of capital. So past investments increase the value of land for both business and residential uses.

Location r is used for business purposes at time t, given that $k_t(r)$ units of business capital are installed, if

$$R(k_t(r), z^t(r), w^t(r)) \geq q(k_t(r), w^t(r)) \tag{26}$$

In this case the fraction of land used for business purposes, $\theta_t(r)$, is such that $\theta_t(r) > 0$. $\theta_t(r) = 1$ if the inequality is strict. If

$$R(k_t(r), z^t(r), w^t(r)) < q(k_t(r), w^t(r)), \tag{27}$$

installed business capital at r will be transferred to residents. In this case $\theta_t(r) = 0$.

Location r is used for residential purposes at time t, given that $k_t(r)$ units of residential capital are installed, if

$$q(k_t(r), w^t(r)) \geq R(k_t(r), z^t(r), w^t(r)). \tag{28}$$

So $\theta_t(r) < 1$, and $\theta_t(r) = 0$ if the inequality is strict. Alternatively, if

$$q(k_t(r), w^t(r)) < R(k_t(r), z^t(r), w^t(r)) \tag{29}$$

then all the residential capital will be transferred to the firm that will buy land at that particular location. In this case $\theta_t(r) = 1$.

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There is competition for land at every location and point in time. Hence land rents will be equal to either the willingness to pay of firms or of residents. The standard reason is that if this is not the case, a particular firm may want to pay more than $q\left(k_t\left(r\right), w^t\left(r\right)\right)$ but less than $R\left(k_t\left(r\right), z^t\left(r\right), w^t\left(r\right)\right)$ for land at r. However there are other potential entrants that would then outbid this firm since they are willing to pay more for that location. That is, there is competition for locations among firms, residents, and between firms and residents. Since business or residential capital can be purchased by anyone, and there is no cost of adapting capital to different uses, all firms (residents) will have identical willingness to pay for land with a certain amount of installed capital.

In the formulation above we are assuming that if land switches sector at time t in location r,

$$R\left(k_t\left(r\right), z^t\left(r\right), w^t\left(r\right)\right) = q\left(k_t\left(r\right), w^t\left(r\right)\right),$$

so there are no profits to be made by either residents or firms. This is important since the problem becomes much more complicated if we have to keep track of the potential gain of selling land to the other sector. The assumption is natural if both functions R and q are continuous in all variables and these variables evolve continuously through time. The fact that time is discreet may imply that gains could be realized from one period to the next, we take these gains to be small and ignore their impact. They would certainly not arise in a continuous time framework. Also notice that the assumption is completely innocuous in steady state since in steady state land does not switch use.

Given the equilibrium land use structure , $\theta\left(r\right)$, capital accumulation at each location in the city, $r \in [-S, S]$, is given by

$$k_{t+1}\left(r\right) = k_t\left(r\right) + \theta(r)i_t^B\left(r\right) + \left(1 - \theta\left(r\right)\right) i_t^R\left(r\right). \tag{30}$$

2.4 Commuting costs and labor mobility

Commuting is costly in time. All agents are endowed with one unit of time each period. They use time working and commuting. An agent living at s and working at r has to commute the distance $|r - s|$ twice daily. The time he has available for work is given by

$$e^{-\kappa|r-s|} \tag{31}$$

hours of labor at location r.

Free mobility of labor across locations implies that wages and expenditures (both denoted by w depending on whether we are in a business or residential area) have to satisfy a no arbitrage condition, or all t and $s, r \in [-S, S]$

$$e^{-\kappa|r-s|} w_t(s) \leq w_t(r) \leq e^{\kappa|r-s|} w_t(s). \tag{32}$$

One of the week inequalities holds with equality whenever some agents commute between r and s. In these cases the condition says that expenditures are given by wages minus commuting costs.

2.5 Equilibrium

We are ready to define a perfect foresight competitive equilibrium in a city.

Definition 1 *Given an initial distribution of business and residential capital, $k_0(r)$, $r \in [-S, S]$, an equilibrium is a sequence of piecewise continuous functions θ_t, and k_t, and a collection of sequences $\left(z, n, i^B, N, c, \ell, i^R, w, R, Q, q, H\right)$ of continuous functions, all on $[-S, S]$, such that for all r and all $t = 0, 1, ...,$*

1. $w_t(r)$ satisfies (32),

2. $n_t(r) = \hat{n}_t\left(k_0(r), z^t(r), w^t(r)\right)$, $i_t^B(r) = \hat{i}_t^B\left(k_0(r), z^t(r), w^t(r)\right)$,
   and $R_t(r) = R\left(k_0(r), z^t(r), w^t(r)\right)$,

3. $Q^0(r) = \hat{Q}^0(k_0(r), w^0(r))$, $c_t(r) = \hat{c}_t\left(k_0(r), \hat{Q}^t(r)\right)$,
   $\ell_t(r) = \hat{\ell}_t\left(k_0(r), \hat{Q}^t(r)\right)$, $i_t^R(r) = \hat{i}_t^R\left(k_0(r), \hat{Q}^t(r)\right)$,
   $N_t(r) = 1/\ell_t(r)$, and $q_t = q(k_0(r), w^t(r))$,

4. $0 \leq \theta(r) \leq 1$ and (26)-(30) are satisfied,

5. $H_t(0) = H_t(S) = 0$, where

$$\frac{dH_t(r)}{dr} = [\theta_t(r) n_t(r) - (1 - \theta_t(r) N_t(r))] + \kappa H_t(r) \quad \text{if } H_t(r) \geq 0, \quad (33)$$

$$\frac{dH_t(r)}{dr} = [\theta_t(r) n_t(r) - (1 - \theta_t(r) N_t(r))] - \kappa H_t(r) \quad \text{if } H_t(r) < 0, \quad (34)$$

and

6. z_t, θ, and n satisfy

$$z_t(r) = \Phi \int_{-S}^{S} e^{-\Phi|r-s|} s\theta_t(s) n_t(s) \, ds. \quad (35)$$

Conditions 5 is the equilibrium condition in the labor market. It guarantees that the quantities of hours of work supplied and demanded in the city are equalized after accounting for commuting costs. Condition 6 specifies the production externality. Productivity at location r is a weighted average of employment at all other locations in the city. The weights are exponential and decrease with distance.

To guarantee existence of an equilibrium allocation in this economy, we need to impose several restrictions on preferences and technology. The assumptions parallel those in Assumption A in Lucas and Rossi-Hansberg (2002). Here we use the following assumptions:

Assumption A *(i) U and f are twice continuously differentiable functions and g is a continuously differentiable function. (ii) U and f are strictly increasing in all arguments, (iii) $f_{nn} < 0$, $f_{nk} > 0$, $f_{kk} < 0$, $g_z > 0$, and (iv)*

$$\lim_{n \to \infty} \frac{\partial g(n) f(n, k)}{\partial n} = 0. \quad (36)$$

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The proofs of existence parallel the ones in Lucas and Rossi-Hansberg (2002) so we omit them here.

2.6 Unanticipated shocks

Suppose the nature of shocks is to destroy capital in certain areas of the city. If shocks are *unanticipated* and are *expected never to happen again*, the framework above is rich enough to study the effect of shocks on urban structure. Two types of results are plausible. If the steady state of a city is unique, the shock leads to transitional effects but no long term effects. The city will return to its previous equilibrium even though it may take some time and there may be substantial costs related to the transition. If the city has several steady states, the shock may change the steady state to which the city converges to. This second scenario may be much more costly in terms of welfare.

The problem is much more difficult if the shocks are unanticipated, but after the shock agents believe that the probability of some shock happening again is positive and bounded away from zero in all (or some) areas of the city. In this case, the problem above has to be modified to deal with the agent's expectation about the future, given that the possibility of shocks implies that there is aggregate uncertainty. In order to set up a parallel version of the model in which we can address this issue, assume that prior to the original shock, agents believed that the probability of a shock was zero. Everything in the economy was deterministic. Another way to express this is that the probability distribution over states of nature was degenerate. Let $P(D, r, t)$ be the probability density that a shock destroys a fraction D of the capital stock at location r in period t. $P(D, r, t)$ does not have to be continuous with respect to D. It may have some mass at particular points, for example no attacks or complete destruction. For simplicity, assume that D is either 1 or 0. Since P is a density function $P(0, r, t) + P(1, r, t) = 1$.

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Prior to the initial shock $P(0, r, t) = 1$ all r, t. Namely, the probability of an attack at any location in the city at any point in time is zero. Suppose an attack destroys the structures at all locations with capital level larger than K_0. We will assume that the probability distribution is updated in a way that assigns probability $p(k_t(r), K_0)$ of a future attack at locations with capital $k_t(r)$. We assume that p is an increasing, continuous and differentiable function of capital at that particular location, as well as of the threshold K_0.[4] We assume that this probability function is updated *once and for all* after the attack. That is, suppose at time t_0 a shock destroyed the capital stock in all locations such that $k_t(r) \geq K_0$. That is

$$D_{t_0}(r) = \begin{cases} 1 \text{ if } k_t(r) \geq K_0 \\ 0 \text{ if } k_t(r) < K_0 \end{cases} \tag{37}$$

Then, for all $t \leq t_0$, $P(1, r, t) = 0$ all r, and for all $t > t_0$,

$$P(1, r, t) = p(k_t(r), K_0) \equiv \bar{p}(k_t(r)). \tag{38}$$

A firm at time t_0 given capital at location r in period t_0, $k_{t_0}(r)$, maximizes it's expected output minus labor and investment costs

$$R\left(k_{t_0}(r), z^{t_0}(r), w^{t_0}(r)\right) \tag{39}$$

$$= \max_{\{n_j, k_j\}_{j=t_0}^{\infty}} E_{t_0}\left[\sum_{j=t_0}^{\infty} \beta^{j-t_0}\left[g(z_j(r)) f(n_j(r), k_j(r)) - w_j(r) n_j(r) - I(i_j(r))\right]\right]$$

subject to

$$k_{t+1}(r) = (1 - \delta)(1 - D_t(r)) k_t(r) + i_t^B(r) \text{ all } t \geq t_0. \tag{40}$$

The expectation above uses the probability distribution $P(D_t(r), r, t)$. Capital stocks at t_0 are given by one minus the percentage of installed capital destroyed, $1 - D_{t_0}(r)$,

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[4] $p_1(k, \cdot) > 0$ captures the idea that large buildings are an attractive target for terrorist since they maximize destruction. However, large buildings also imply lower per person, or per unit of capital, security costs (see Eckstein and Tsiddon 2004). We assume throughout the paper that the first effect dominates .

times the steady state capital stock. Namely, $k_{t0}(r) = (1 - D_{t_0}(r)) \bar{k}(r)$ all $r \in [-S, S]$.

Suppose that there are no adjustment costs so that $I(i) = i$. In this case we know that agents will adjust their capital stocks immediately to the new steady state level, and therefore they will maintain the new steady state capital level forever. The steady state capital level in this case is given by

$$\frac{\beta \left[g(\bar{z}) f_k(\hat{n}, \bar{k}) \right]}{(1 - \beta + \beta\delta)} - \frac{\beta \left[\bar{p}'(\bar{k}) \bar{k}(1 - \delta) + \bar{p}(\bar{k})(1 - \delta) \right]}{(1 - \beta + \beta\delta)} = 1. \tag{41}$$

Hence, since $\bar{p}' > 0$ (a larger building implies that the probability of an attack increases) the second term is negative. Assuming f_k is a decreasing function of the capital stock, the capital stock in steady state after the shock, given $\bar{z}$ and θ, is smaller than before the shock. Of course, in equilibrium $\bar{z}$ and θ are not given and so we can not make this a general statement. The basic problem is that the location of business and residential areas could change after the shock, thereby altering productivity, and the amount firms are willing to pay for land at locations close to the areas that became residential with the terrorist threat. Without adjustment costs the willingness to pay for land of a firm at location r, in steady state, is given by

$$R(\bar{k}, \bar{z}, \bar{w}) = \frac{1}{1 - \beta} \left[g(\bar{z}) f(\hat{n}, \bar{k}) - \bar{w}\hat{n} - \left[\bar{p}(\bar{k}) I(\bar{k}) + (1 - \bar{p}(\bar{k})) I(\delta\bar{k}) \right] \right]. \tag{42}$$

If in contrast the I function is convex, we know that agents will adjust slowly their capital level after a shock. The particular path of capital stocks in a given location will depend on the realized set of shocks.

The problem of a consumer at time t_0, that lives at location r with capital $k_{t_0}(r)$, becomes, after the attack,

$$\min_{\{c_j, \ell_j, k_j\}_{j=t_0}^{\infty}} E_{t_0} \left[\sum_{j=t_0}^{\infty} \beta^{j-t_0} \left[c_j(r) + Q_j(r)\ell_j(r) + I\left(i_j^R(r)\right) \right] \right] \tag{43}$$

subject to

$$\bar{u} \leq E_{t_0} \left[\sum_{j=t}^{\infty} \beta^{j-t_0} \left[U \left(c_j \left(r \right), \ell_j \left(r \right), k_j \left(r \right) \right) \right] \right], \qquad (44)$$

$$k_{t+1} \left(r \right) = \left(1 - \delta \right) \left(1 - D_t \right) k_t \left(r \right) + i_t^R(r), \text{ all } t \geq t_0. \qquad (45)$$

Again suppose that there are no adjustment costs so that $I(i) = i$. The steady state capital level in residential areas is then given by

$$\frac{\beta}{1 - \beta + \delta\beta} \left[\frac{U_k \left(\hat{c}, \hat{\ell}, \bar{k} \right)}{U_c \left(\hat{c}, \hat{\ell}, \bar{k} \right)} \right] - \frac{\beta \left[\bar{p}' \left(\bar{k} \right) \bar{k} \left(1 - \delta \right) + \bar{p} \left(\bar{k} \right) \left(1 - \delta \right) \right]}{\left(1 - \beta + \beta\delta \right)} = 1. \qquad (46)$$

The residential bid rent at location r in steady state, without adjustment costs, is given by the value of $\hat{Q}$ that solves

$$\frac{\bar{w}}{1 - \beta} = \frac{\hat{c} \left(\bar{k}, \hat{Q} \right) + Q\hat{\ell} \left(\bar{k}, \hat{Q} \right) + \left[\bar{p} \left(\bar{k} \right) I \left(\bar{k} \right) + \left(1 - \bar{p} \left(\bar{k} \right) \right) I \left(\delta \bar{k} \right) \right]}{1 - \beta}, \qquad (47)$$

and the willingness to pay for land is given by

$$q \left(\bar{k}, \bar{w} \right) = \frac{\hat{Q} \left(\bar{k}, \bar{w} \right)}{1 - \beta}. \qquad (48)$$

Assuming a concave utility function so that $U_k \left(\hat{c}, \hat{\ell}, \bar{k} \right) / U_c \left(\hat{c}, \hat{\ell}, \bar{k} \right)$ is a decreasing function of $\bar{k}$, we obtain the following result.

Proposition 1 *Without adjustment costs, the steady state capital level in residential areas is higher before than after a shock that destroys part of the capital in the city.*

Proof. Envelope theorem plus $U_k \left(\hat{c}, \hat{\ell}, \bar{k} \right) / U_c \left(\hat{c}, \hat{\ell}, \bar{k} \right)$ is a decreasing function of $\bar{k}$. ∎

Other results can be proven in a similar way. For example, we know that if $\bar{p}'' > 0$, $\beta \left[\bar{p}' \bar{k} \left(1 - \delta \right) + \bar{p} \left(1 - \delta \right) \right]$ increases with $\bar{k}$. Hence locations with larger amounts of capital before the shock will reduce, in absolute terms, their amount of capital more

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than locations with a lower amount of capital before the shock. The direct effect on $\beta \left[\bar{p}' \bar{k} \left(1 - \delta \right) + \bar{p} \left(1 - \delta \right) \right]$ is further emphasized by the fact that $\hat{n}$ is an increasing function of capital, and so z is an increasing function of the absolute level of capital at all locations. This argument is valid for both business and residential areas. Hence capital densities in the city will become flatter after the shock, in the absence of adjustment costs. We formalize the result in the next proposition.

Proposition 2 *Without adjustment costs, the range of steady state capital levels across locations within the city decreases after the shock. That is,* $\max_{s \in [-S,S]} k(s) - \min_{s \in [-S,S]} k(s)$ *decreases after a shock is realized.*

Proof. If $\bar{p}'' > 0$, $\beta \left[\bar{p}' \left(\bar{k} \right) \bar{k} \left(1 - \delta \right) + \bar{p} \left(\bar{k} \right) \left(1 - \delta \right) \right]$ increases with $\bar{k}$ which proves the result for residential areas. This, the first order condition with respect to capital, $\partial \hat{n} / \partial \bar{k} > 0$, and $\bar{z}$ an increasing function of $\hat{n}$ at all locations, yields the result for business areas. ■

3. POLICY DESIGN

There are two relevant policy questions in this setup. The first one arises since we are using externalities to create agglomeration in the city. This implies that there is room for policies that make firms internalize the positive externality they impose on other firms. This type of policies has been studied in a framework without capital in Rossi-Hansberg (2004). The exact same logic applies in this setup, a location specific labor subsidy paid to firms that takes the form

$$\tau(r) = \begin{cases} \Phi \int_0^S e^{-\Phi|r-s|} s\theta(s) g'(z(s)) f(n(s), k(s)) dr & \text{if } \theta(r) > 0 \\ 0 & \text{otherwise} \end{cases}, \qquad (49)$$

implements the optimal allocation as an equilibrium, where the optimum is defined as the allocation that maximizes total rents in the city.

The second policy question that arises in this setup is whether government policy can eliminate the effect of a terrorist threat. Before answering this question, it is important to notice that the reaction of agents to the terrorist threat is optimal from the private point of view. An agent invests less, in general, because of the possibility of future destruction of their capital stocks and because increasing their capital stock implies a larger probability that a future attack will target their business or residence. This reduction is optimal for private agents given their assessment of the probability of a future attack. Any policy on top of the one presented above, necessary because of the externality, will distort private incentives and hence be suboptimal. However, if the government has better information than private agents and believes that the probability of a future attack is zero, and cannot convince agents that this is the case, it could potentially eliminate the distortion created by the agents wrong (according to the government) beliefs. In this context, we can ask what instruments the government would need to use in order to eliminate the effect of a terrorist threat on the city.

Since the only dynamic aspect of the model is capital accumulation at each location, a natural instrument to eliminate the effect of a terrorist threat on the city is a capital subsidy. In fact, we will show below that a subsidy paid to residents and firms will eliminate the effect of the terrorist threat. Let us first provide some intuition. The cost of a future threat is, on one hand, that all the installed capital at a particular location may be destroyed. This happens with probability $\bar{p}\left(k_t\left(r\right)\right),$ if $k_t\left(r\right)$ units of capital are installed at location r. Hence the expected value of the loss next period is given by this probability times the cost of installing the desired amount of capital next period, $\bar{p}\left(k_t\left(r\right)\right)I\left(k_{t+1}(r)\right).$ On the other hand, in the absence of an attack we have to replace the depreciated capital. The expected cost of doing this is given by $\left(1-\bar{p}\left(k_t\left(r\right)\right)\right)I\left(i_t\left(r\right)\right).$ Compare these costs with the cost of replacing depreciated capital in the absence of a terrorist threat, $I\left(i_t\left(r\right)\right).$ We need to design a subsidy that depends on the amount of capital installed as well as investments, such that

every period these costs are the same with or without the attack. That is, a subsidy $\pi\left(k_t\left(r\right), i_t\left(r\right)\right)$ such that

$$\bar{p}\left(k_t\left(r\right)\right) I\left(k_{t+1}(r)\right) + \left(1 - \bar{p}\left(k_t\left(r\right)\right)\right) I\left(i_t\left(r\right)\right) - \pi\left(k_t\left(r\right), i_t\left(r\right)\right) = I\left(i_t\left(r\right)\right), \quad (50)$$

where $i_t\left(r\right) = \theta\left(r\right) i_t^B\left(r\right) + \left(1 - \theta\left(r\right)\right) i_t^R\left(r\right)$, all r.

In steady state, capital levels are given by $\bar{k}(r)$, so we need a subsidy that only depends on the capital installed, namely

$$\bar{\pi}\left(\bar{k}(r)\right) = \bar{p}\left(\bar{k}(r)\right)\left[I\left(\bar{k}(r)\right) - I\left(\delta\bar{k}(r)\right)\right]. \quad (51)$$

We formalize this result in the next proposition.

Proposition 3 *A capital subsidy for firms and residents of the form*

$$\pi\left(k_t\left(r\right), i_t\left(r\right)\right) = \bar{p}\left(k_t(r)\right)\left[I\left(\left(1 - \delta\right) k_t(r) - i_t\left(r\right)\right) - I\left(i_t\left(r\right)\right)\right] \quad (52)$$

eliminates the effect of a terrorist threat on the city.

Proof. For business areas, impose the subsidy in the problem presented in (39) each period and substitute the evolution equation for capital in (40), to obtain the problem in (2). A parallel argument works for residential areas, impose the subsidy in (43) and substitute the evolution of capital equation (45) to obtain the problem presented in (11). ∎

Notice that we have been silent about the source of funds necessary to finance this policy, thereby implicitly assuming that it can be financed using external funds or lump-sum taxes. We have mentioned several caveats of proposing this capital subsidy. First of all the government or the authority imposing the tax must have better information that the public (that an attack will never happen again), and should be unable to communicate this information credibly. This seems plausible given that some of the relevant information may be classified. If all these conditions

are satisfied, then imposing a subsidy of this sort would be beneficial. A different question is whether the public will be in favor of the subsidy or not. As we will show with numerical exercises in the next section, land rents are not everywhere lower with than without the terrorist threat. Some landlords will lose from this type of policy, although many will win (if in fact another attack is never realized). The important point is that the model implies that there will not be universal support for such a policy, even if we do not consider the cost of the resources used for the subsidy. Proposition 3 also implies that a terrorist threat has the same effect as a tax on capital, where the resulting revenue is thrown away.

4. NUMERICAL RESULTS AND SPECIFICATION

To compute numerical simulations of the model we need to specify the production, adjustment costs, and utility functions. We will specify the production function as Cobb-Douglas with constant returns to scale. Namely,

$$g(z)f(n,k) = Az^{\gamma}n^{\alpha}k^{\rho} \tag{53}$$

where $\alpha + \rho < 1$. The adjustment cost function is given by

$$I(i) = i + \iota i^2 \tag{54}$$

The utility function is given by

$$U(c, \ell, k) = c^{\mu}k^{\nu}\ell^{1-\mu-\nu}. \tag{55}$$

We also need to take a stand on the empirical specification of the probability of a next period attack, given that a past shock destroyed all the capital at locations that had more than K_0 units of capital. The function we use in all the exercises below is given by

$$\bar{p}(k) = \min\left[1, \hat{p}\frac{k}{K_0}\right], \tag{56}$$

where $\hat{p} > 0$ is a scalar that determines the likelihood of a future attack. That is, the probability of a future shock in a location with capital stock k is a linear and increasing function (bounded at one) of the ratio between the installed capital stock and the minimum capital stock installed at locations destroyed in the past.

Given these specifications and once the corresponding parameter values are determined, together with the reservation utility level and the production externality parameters, we can compute the steady state allocation in a city. This implies solving for a collection of functions, all in $[-S, S]$. The functions that determine the equilibrium allocation are productivity, capital stocks, employment density, consumption, residential density, residential lot size, land use, wages, and land rents. Given a function of capital stocks in the city as initial condition, we can also solve for the transition of the city to a steady state. In the following two subsections we compute and discuss equilibrium allocations for different parameter values.

4.1 Steady state

The first exercise compares the steady state allocations without an attack threat with the steady state after an actual attack was realized. We do this assuming that $I(i) = i$, that is, that there are no adjustment costs. In this case there is no transition. After the attack is realized the city moves immediately to the new steady state. The first exercise is for low commuting costs, $\kappa = 0.001$. As we have argued before, equilibrium allocations in this type of framework depend crucially on the size of commuting costs. We will vary κ and other parameters in order to understand how the structure of the city reacts to terrorist attacks. We will also vary other parameters that can be potentially affected by the threat of terrorist attacks, in particular the externality parameter Φ. We start with an exercise in which all the fundamental parameters in the economy are the same before and after the attack. The attack destroys part of the capital of the city, in particular, it destroys capital in locations

that had more than K_0 units of capital. Since there is no transition in these examples, locations were capital was destroyed are not different than locations where capital was not destroyed. The ex-post probability of another attack will depend on how much capital was installed in a particular location in the new steady state and not on which parts of the city were originally destroyed.

The parameter values that we use in the exercise below are given by

A	α	ρ	γ	ι	$\bar{u}$	μ	ν	β	δ	Φ	S	dr
2	.7	.25	0.04	1	1	.6	.3	.9	.1	5	10	.01

,

we also let $K_0 = 17$ and $\hat{p} = 0.1$. In all the exercises we maintain the assumption that the city events are not large enough to change the price of the good produced or the reservation utility in the country. Studying those effect would require us to embed this model in a model of a system of cities in a country.

The four graphs in Figure 1 illustrate the steady state effect of the new threat of terrorist attacks on a city with these fundamentals. Throughout this section the thin lines in the figures present the allocation without terrorist attacks and the thick lines present the allocation after an attack was realized and so there is a threat of future attacks. The figure in the upper left corner presents land values. The comparison between business and residential bid rents determines the equilibrium land use structure and land rents (upper envelope of business and residential bid rents). Without a terrorist threat, given low commuting costs $\kappa = 0.001$, the resulting city is monocentric. However, land values before the attack do not decline monotonically from the center. The reason is that land services can be substituted for capital services and it is more profitable to do that closer to residential areas. The equilibrium land use structure consists of a business sector from the center of the city that expands about 7.5 miles from the center in both directions, and residential areas from there until the boundary of the city at $S = 10$ (or -10) The capital stocks associated

with this allocation are presented in the upper right corner of Figure 1. The capital structure of the city follows a similar pattern than land rents. Notice that even though residents are willing to invest more in capital at the center, firms are willing to pay more for these locations.

Figure 1: Equilibrium allocation and prices, kappa = 0.001, Ko = 17
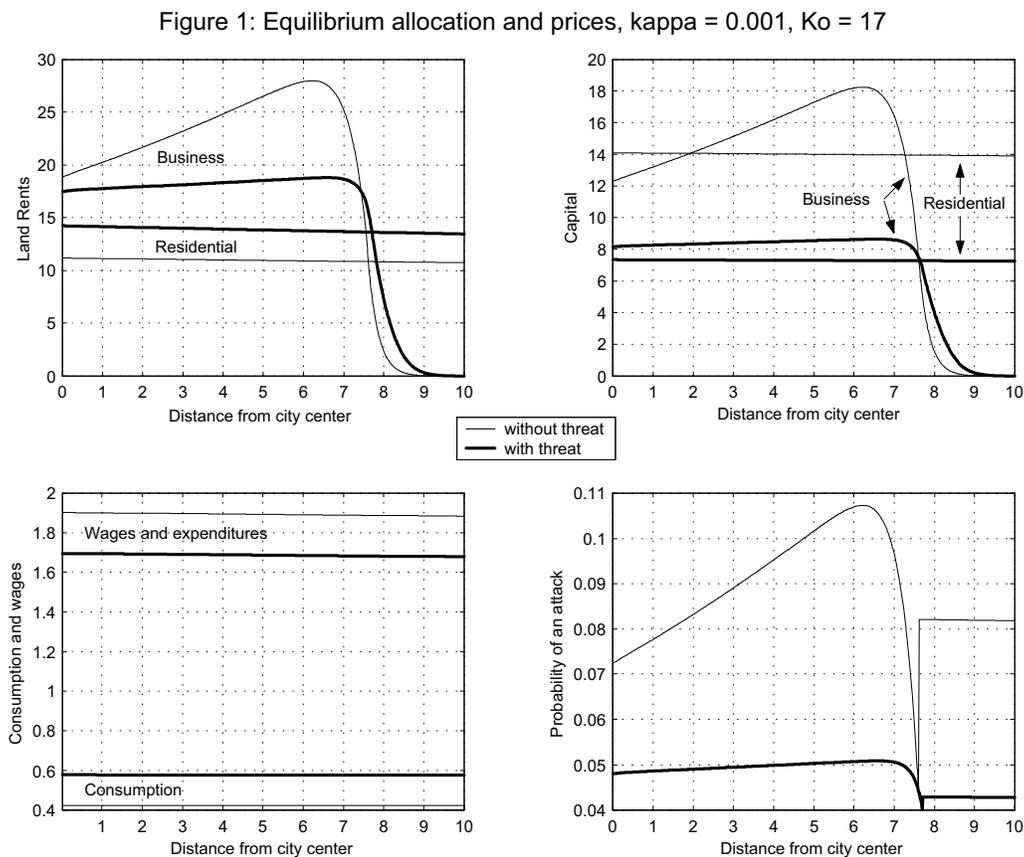
An attack destroys all the capital installed in locations with $k_t(r) > K_0$. This implies that after the attack there is a threat of future attacks that depends on the height of buildings that were previously destroyed. The probability of destruction in the future in both equilibria (before and after the attack) is presented in the lower east panel of Figure 1. The threat of future attacks implies that the allocation without threat is no longer an equilibrium, the reason is that the probability of a future attack is too large. The new allocation has less capita installed everywhere in the

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city and the range of the capital stocks (as well as rents) is smaller, as we knew from Propositions 1 and 2. Notice that capital is also lower in business areas, the reason is that the land use structure changes only slightly. The land use structure with threat is also monocentric with a business area that starts at the city center and extends for 7.6 miles in both directions. Business areas expand relative to residential areas -firms substitute capital for land-, although this effect is rather small given the very important exodus from the city. The equilibrium after the attack has only 88% of the residents in the equilibrium allocation without a terrorist threat. Land rents in business areas are in general smaller, but not at all locations. This contradicts the widely held view that land rents will decrease uniformly in the city after the shock. As expected, given the effect on rents and capital stocks, wages and expenditures of agents in the city are lower in the new equilibrium, however, agents in the city consume more. The reason is that agents that stay in the city after the attack obtain the same utility as agents that were in the city before the attack. In order for them to receive the same utility as before, they have to compensate the lower amount of capital in their houses with more land and more consumption. Therefore, land rents in residential areas are *higher* with the terrorist threat. On one hand, agents consume more land (even though land rents are higher), on the other they consume more goods. In this sense, a terrorist threat is similar to a location specific tax on installed capital, as we have shown before.

The next set of figures, grouped in Figure 2, illustrate parallel results for $\kappa = 0.005$. There are now three residential areas in the city. One is located at the east boundary, a second at the center, and a third at the west boundary. There are two business sectors, each of which has two important concentrations of businesses. An increase in transportation cost from $\kappa = 0.001$ to $\kappa = 0.005$ changed the city dramatically. However, the presence of a terrorist threat has again a modest effect on city structure. The residential area at the center becomes larger with the threat and the one at the

boundary smaller. Population in the city decreases by only 2%.

In the allocation illustrated in Figure 2, the capital stock in business areas is in general smaller with the threat than without it, except for locations near to the outer limit of the business sector and in the middle of the business areas. This example demonstrates that even though capital in general will be smaller in the city, some areas may see investments increase after the shock. The decrease in capital is evident throughout residential areas. Rents are not uniformly lower in business areas, and they are uniformly *higher* in residential areas. This is the effect of investing less in capital in residential areas. In business areas, this effect is less important since the positive production externalities make the reduction in capital investments smaller.

Figure 2: Equilibrium allocation and prices, kappa = 0.005, Ko = 17
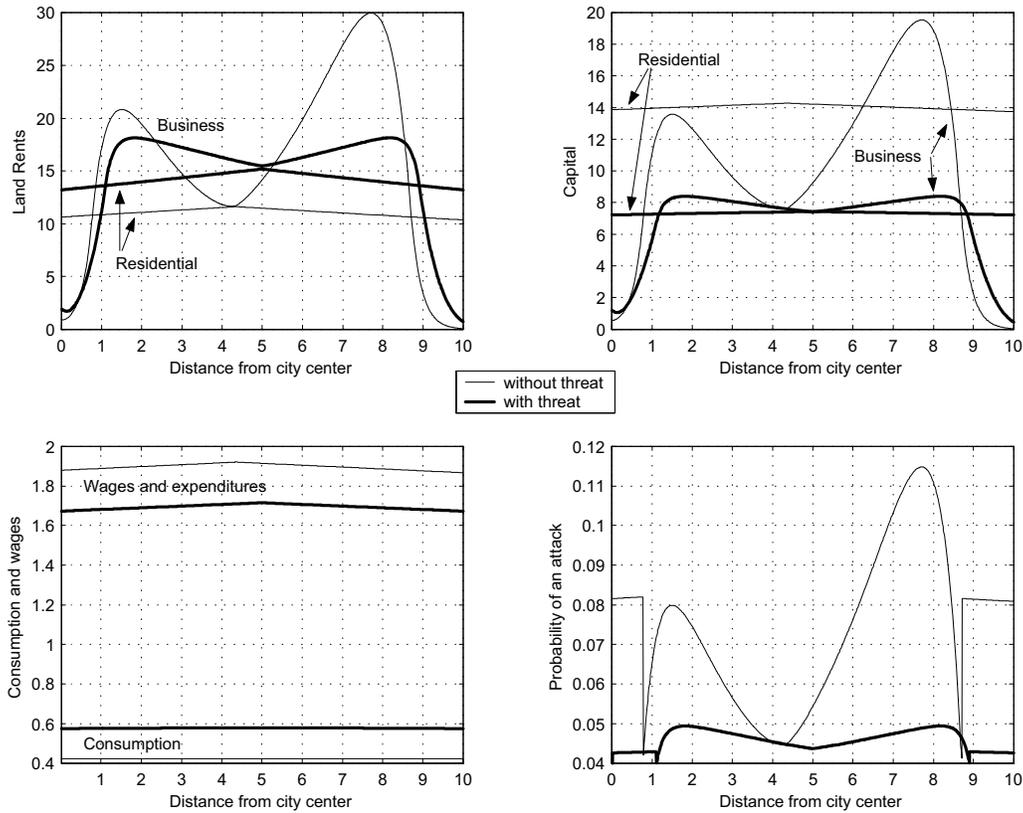
Combining Figure 1 and 2 one can also talk about the effect of an attack that not

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only changes the agent's probability assessment of future attacks, but also changes the actual fundamentals of the economy. In particular, the shock may changes transport cost per mile. The reason this may happen is that the potential of an attack increases security checks, as well as the risk associated with using public transportation. However, as we have argued above, it may also decrease densities in the city, which potentially (although not in this model) may lead to reductions in transport costs. In any case, we can study the effect of an attack that increases commuting cost as well as the probability of future attacks. This means comparing the thin lines in Figure 1 with the dark lines in Figure 2. In contrast with the other cases, the change in the city structure is very important. A residential area appears at the center and the agglomeration of firms and capital investment change substantially too. Land values would, however, not be lower everywhere and residential capital and land rents would still be higher with the threat. The new city would have 84% of the population of the city without the threat. These changes are much more dramatic than in the case with constant commuting costs. The exercise was computed for a 500% increase in transport costs which is very large and probably unrealistic. Our goal is to illustrate the presence of qualitative effects, not to do a quantitative assessment of the magnitude of these effects.

Terrorism may also affect the spatial scope of production externalities. The increased obstacles for human interaction imply that economic activity in a particular location will benefit, via production externalities, only from economic activity located very close by. The decay in the benefits received from economic activity at other locations is governed by the parameter Φ. Notice that in Equation (35) Φ enters in the exponential discount and multiplying. Hence by changing Φ we are not reducing average externalities, only their spatial decay. The idea is that the scope of externalities may decrease with a terrorist threat but agents located nearby will therefore interact more, which in average compensates the lost interaction with employees located far

away. All the exercises above kept $\Phi = 5$. We also computed exercises for $\Phi = 10$ and $\Phi = 30$. The results differ only modestly from the ones presented in Figure 1. Output in the city, without a terrorist threat, increases by 0.9% for $\Phi = 10$ and 1.24% for $\Phi = 30$ relative to the exercise in Figure 1. We conclude from these exercises that this channel is of only modest importance and so do not present the complete set of results here.

In both examples discussed above, output decreases because of the threat of a terrorist attack: 21% for $\kappa = 0.001$ and 12% for $\kappa = 0.005$. Hence, if an attack does not materialize ever again, there are potentially important gains from imposing a set of capital subsidies with the form described in the previous section. In these two examples, the cost of the policy is lower than the gains implied by eliminating the effect on the city of a terrorist threat (if no future attacks are realized). The fiscal cost of the policy accounts for 46% of the gain for $\kappa = 0.001$ and for 72% of the gain for $\kappa = 0.005$. The calculation assumes that revenue can be collected using lump-sum taxes and so there is no distortion associated with financing the policy. The potential gains from implementing this policy are larger the smaller commuting cost in the city. The reason is that as commuting cost increase, productivity in the city is smaller and so is the average stock of capital in the city (although it may be larger in certain areas). Since terrorist attacks affect the incentives to accumulate capital, and only through capital the rest of the city, loses associated with a terrorist shock are lower the lower commuting costs, and so the effect of policies to prevent these changes are lower as well.

4.2 Transition

The results presented in the previous section assume that there are no adjustment costs in capital investments. If this is the case, the economy will reach a steady state immediately after any shock to the urban capital structure is realized. This is not the

case once we add adjustment costs. Capital will adjust slowly potentially changing the structure of the city as it evolves. In the specification presented above this implies that $\iota > 0$.

Without agglomeration forces, modeled as production externalities in this paper, even in the presence of adjustment costs, investments in a particular location will depend only on city wide or nation wide prices of factors. In this paper we assume that the price of capital is determined in the national or world markets. In this sense, the model is a partial equilibrium framework. Capital at the different locations determines labor demand and therefore wages in the city, which in turn affect investment decisions at all location. It is challenging to compute examples that include this type of equilibrium effects in order to understand the speed of convergence to a new steady state. The reason is that we need to use a "shooting" algorithm in order to solve for the path of capital in *all* areas. The result of such an exercise are, nevertheless, not very interesting, since the total cost of the shock is close to the cost of the installed capital that was destroyed in the attack (properly discounted depending on the speed of convergence). The general equilibrium effects are, for localized attacks, likely to be small, so the effect of wages on capital stocks at other locations will be small as well.

With agglomeration effects, the problem becomes more interesting and much more difficult. Capital destruction in particular areas will imply that locations not directly affected, but that are close to the affected areas, will suffer because of the reduced productivity implied by smaller spillovers from the destroyed areas. Once agglomeration effects are taken into account, the effect of localized capital destruction can be much larger than just the properly discounted cost of reconstruction. The reason is that nearby locations will be affected via a decrease in their productivity caused by lower employment in the destroyed areas. In order to understand this decline in productivity, we need to take a stand on how fast employment, and therefore productivity,

reacts to changes in installed capital. We could assume that labor and productivity adjust immediately, but that there are adjustment cost for capital investments. Another possibility is to say that employment adjust slowly, as does capital. Every period agents predict productivity given the current allocation of labor and make their investment decisions. Frictionless labor mobility is appealing but, at the moment, computationally very costly. It implies solving for capital at all location in the city, and this is only a middle step in solving for the fixed point of productivities. The second exercise is also challenging numerically, but feasible. It implies again solving for capital levels as a function of location as an initial condition, but does not imply solving for the fixed point of productivities at each step. This reduces the computational costs dramatically. In what follows we assume that employment adjusts slowly. We will present one numerical results illustrating the type of transition that this model generates. Our focus is on how destruction in a particular area affects other areas and therefore increases the transition costs.

We start by computing the steady state of a city with adjustment costs and the same parameter values as the ones used to generate Figure 1. Because of the adjustment costs, the steady state allocation is different in levels, but qualitatively has a very similar structure: A business center surrounded by a residential area. The area with the largest stock of capital in the city is located about six miles away from the center. Assume that there is an attack to such a city that destroys all capital structures with more than 5.752 units of capital per unit of land. This results in the destruction of 2% of business capital structures in the city. The destroyed area is 0.13 miles long. Residential areas are not directly affected by the shock. We then compute the transition path of capital stocks to the steady state, assuming that the city will never experience another shock. This implies using the Euler equations from the firm and household problems. We use a shooting algorithm to find the level of capital investments at each location after the shock that puts the city in the transition path

that converges to the original steady state. The first three stages of the evolution of the capital stock are presented in Figure 3.

Figure 3: Transition of business capital stock
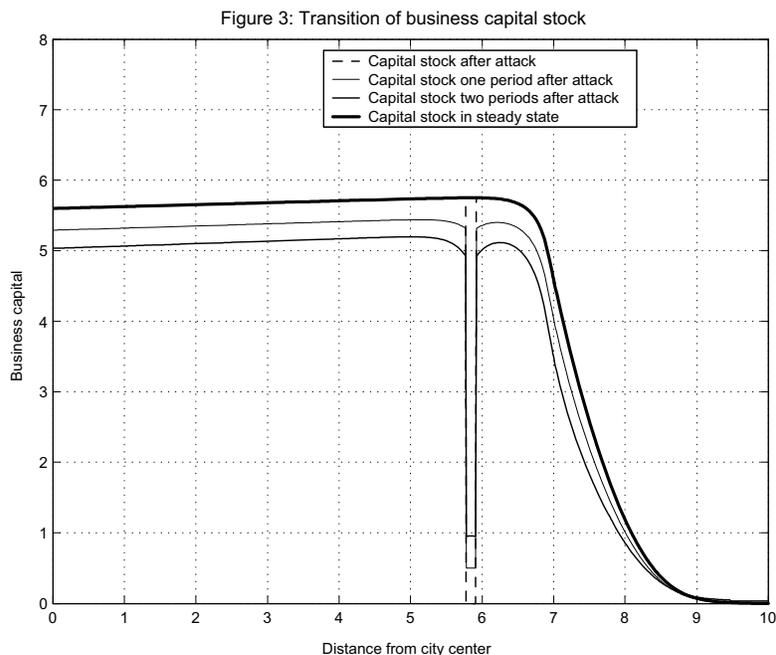
Three effects are evident in Figure 3, first capital in areas near the shock decreases more than in areas far away from it. This is the result of a decrease in productivity at these locations caused by the lack of spillovers from the destroyed areas. Second, the overall level of business capital decreases throughout the city. This is the result of a decrease in the number of people leaving in the city during the transition. Wages in the city also decrease during the first part of the transition. The level of economic activity will eventually recover to reach the original steady state. Third, capital in the destroyed areas starts to build up slowly. In general, the size of the city will shrink during the transition and then will grow to reach the original steady state again. The length of this transition depends, in particular, on the magnitude of the realized shocks and the importance of adjustment costs. Small adjustment costs will lead to fast transitions. This result implies that economist studying the transformation of

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cities after a terrorist act should not interpret the original decline in city size and level of urban economic activity as if this was a permanent change.

The most interesting implications in the model are the ones associated with the effect on areas that were not directly affected by the shock. This is evident in Figure 4 were we present the productivity function, z, in steady state and immediately after the shock (given steady state wages and employment allocations). It is evident from the figure that productivity is affected significantly in areas where capital was not destroyed. The extent of this effect depends, in particular, on the parameter Φ. For higher Φ, the decrease in productivity will be more localized, for lower Φ it will spread out to a larger section of the city.

Figure 4: Productivity before and after the shock
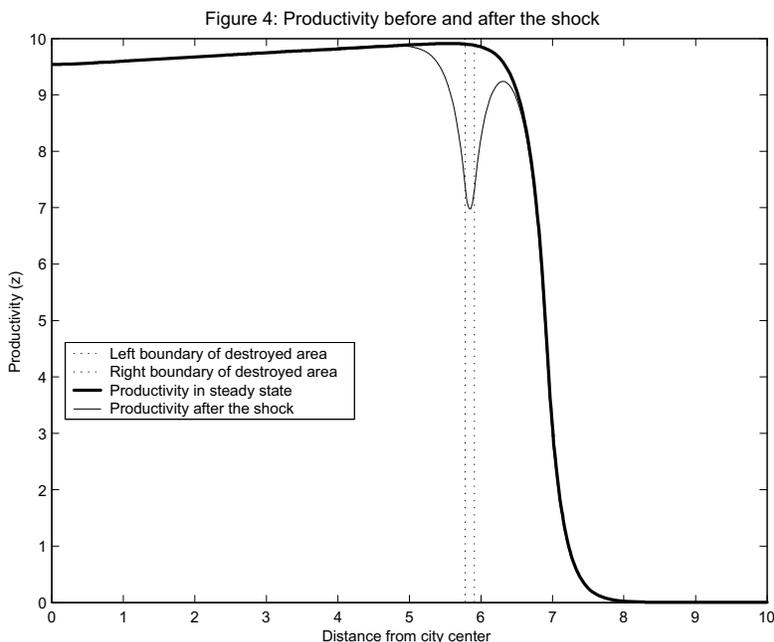
5. CONCLUSION

We presented a theory of the internal structure of cities with physical capital that is suited for the study of urban shocks. We used this theory to analyze how these shocks may result in permanent and temporary effects on urban form. Permanent effects were

the result of changes in the expected returns of capital investments because of the possibility of future shocks, or because of changes in commuting or agglomeration forces as a result of past attacks. The most important conclusions for long run equilibrium allocations are summarized below.

- Terrorist shocks will affect urban structure in the long run, but numerical results show that the distribution of business and residential land will not be affected significantly. In contrast, for the same shocks, the level of steady state economic activity, including production, employment, capital stocks, wages, and land rents will be importantly distorted.

- The range of steady state capital stocks across locations within the city is smaller with the terrorist threat and residential capital stocks will also be uniformly lower.

- Rents do not necessarily have to decline uniformly throughout the city. In all examples computed residential land rents increase, while business land rents decrease in most, but not all, locations.

- The effect of changes in expectations caused by a terrorist attack is equivalent to a location specific tax on capital investments. This implies that a location specific capital subsidy will eliminate the long run effects of an attack. The benefits of this subsidy, net of fiscal costs, are substantial in all numerical examples computed. Since land rents are not lower at all location with than without the terrorist threat, not all landowners will be in favor of the policy. The subsidy is suboptimal unless the government has better information than private agents, that it cannot credibly reveal.

- Changes in commuting costs have important effects on city structure. In particular, increases in commuting costs will lead to a propagation of small business

centers, as previously studied in Lucas and Rossi-Hansberg (2002) and Fujita and Ogawa (1982), among others, in models without capital.

We also analyzed the transitional effects of these shocks in a model with adjustment costs. The main conclusions of this analysis are summarized below.

- Destruction of a particular area reduces productivity in other areas thereby leading to less capital investment in areas located nearby, and to a slow recovery of the destroyed part of the city. This implies much larger costs of transition than the costs associated with the reconstruction of the destroyed areas.

- Capital stocks in destroyed areas will recover slowly, increasing until they reach the steady state levels.

- Cities will first shrink in size to later recover and reach the original level of economic activity. Wages will follow the same dynamic pattern.

In this paper we computed a particular example of this transition. Comparative statics exercises that illustrate how this transition, and in particular the transition costs, change as we change the fundamental parameters of the model are left for further research. These exercises are very interesting, but extremely hard to compute. In order to provide additional analysis on the nature of this transition, we need to take a clear stand on the speed of adjustment of employment, and therefore productivity. The model could be extended to include costs of adjustment for residents moving in and out of the city, as well as for agents changing their business or residential location. These extensions are also left as future research.

One can only finish a paper like this hoping that the analysis in this paper proves helpful to study urban shocks, but, at the same time, hoping that it will be superfluous because an attack of the magnitude experienced on September 11, 2001, never happened again.

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